Statistics in Monte Carlo Analysis
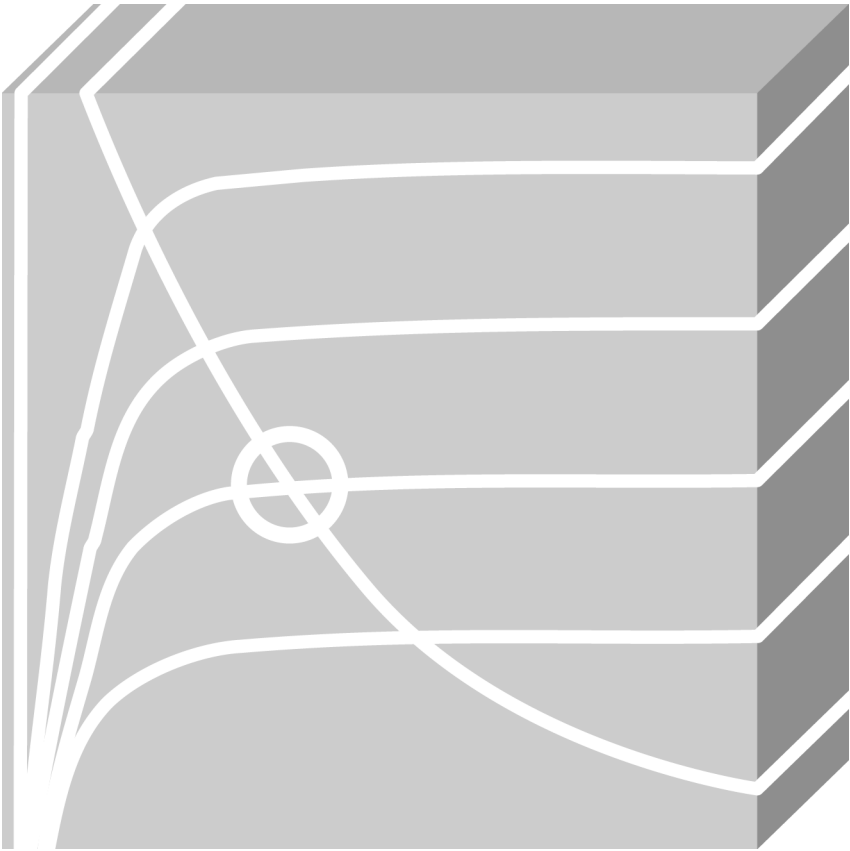
Table of Contents

1   Introduction

The Monte Carlo analysis calculates the circuit response to changes in part values by randomly varying all of the model parameters for which a tolerance is specified. This provides statistical data on the impact of a device parameter's variance. Monte Carlo analysis is frequently used to predict yields on production runs of a circuit. With Monte Carlo analysis, model parameters are given tolerances, and multiple analyses (DC, AC, or transient) are run using these tolerances.

You can display data derived from Monte Carlo waveform families as histograms (using Performance Analysis).


```
n samples    = 100          10th %ile   = 1.71752e-005
n divisions = 10            median      = 2.00674e-005
mean         = 2.01278e-005 90th %ile   = 2.35522e-005
sigma        = 2.38141e-006 maximum     = 2.56332e-005
minimum      = 1.53505e-005 3*sigma     = 7.14423e-006
```
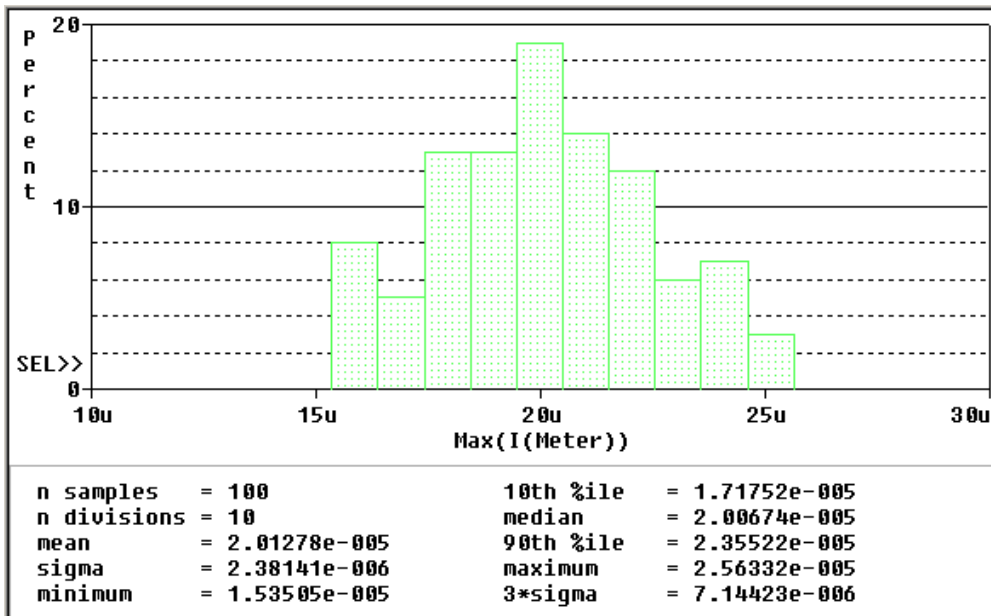
Fig. 1: Histogram of Max(I(Meter))

The MONTE CARLO SUMMARY depended on the collating function e.g. the greatest difference from the nominal run (YMAX) is reported in PSpice output file.

```
****      SORTED DEVIATIONS OF I(V_Meter)  TEMPERATURE =   27.000 DEG C

                   MONTE CARLO SUMMARY

*********************************************************************


Mean Deviation =    28.5690E-09
Sigma          =     2.4445E-06

  RUN                   MAX DEVIATION FROM NOMINAL

Pass   28              5.4934E-06  (2.25 sigma)  higher  at P =    3.6
                       ( 139.92% of Nominal)

Pass   91              5.4736E-06  (2.24 sigma)  higher  at P =    4.8
                       ( 128.52% of Nominal)
```

Fig. 2: MONTE CARLO SUMMARY in PSpice Output File

But how can we understand the statistical data?

2   Demo Project

The following example shows how the performance of a pressure sensor circuit with a pressure-dependent resistor bridge is affected by manufacturing tolerances, using Monte Carlo analysis to explore these effects.

A zero volt voltage source is commonly used as a current meter in PSpice. A zero ohm resistor is not allowed.
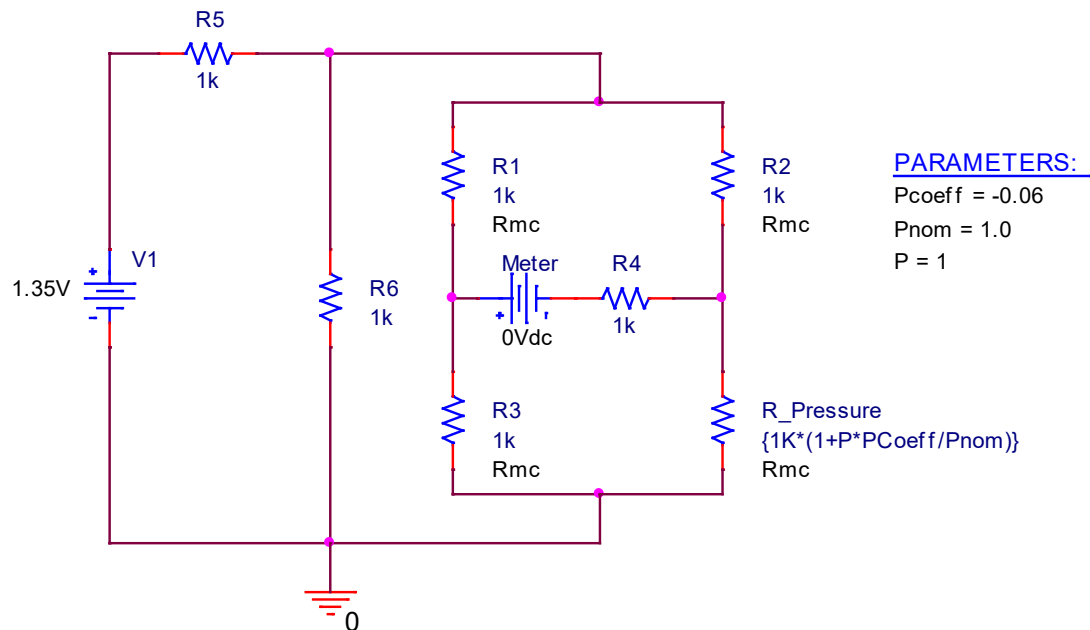
In the above circuit, the model with the following definition

```
.model Rmc RES R=1 DEV=2% LOT=8%
```

is assigned to resistors R1, R2, R3 and R_Pressure while R4, R5 and R6 are normal resistors without tolerance.
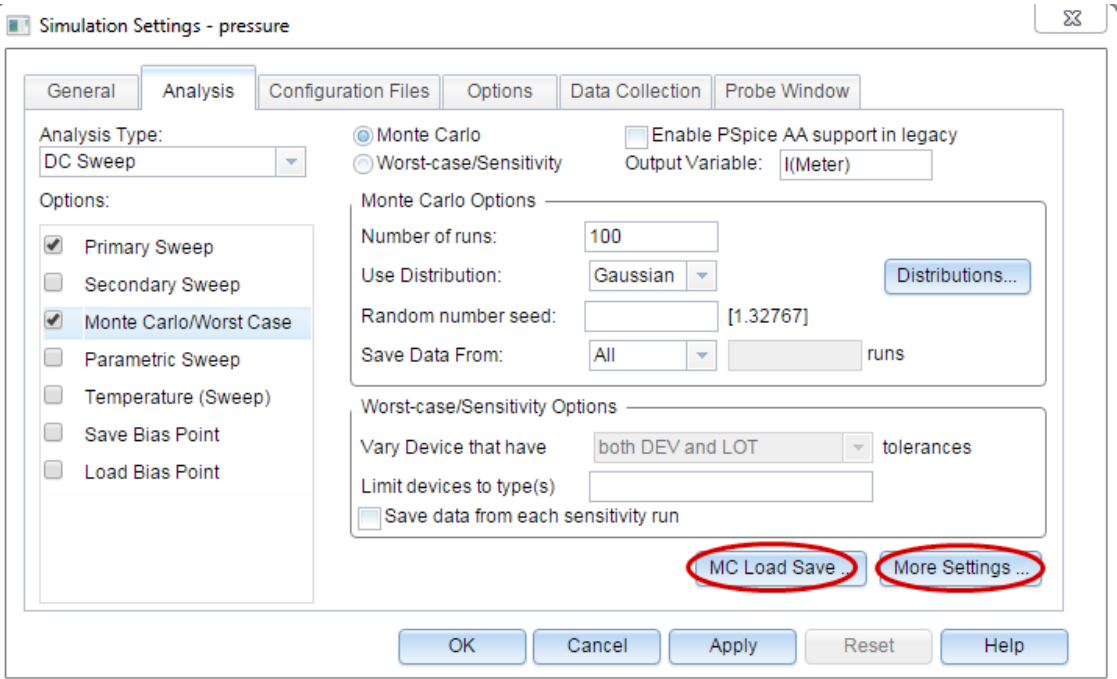
Note

- Both tolerances DEV and LOT are available for each model parameter. Device tolerances, denoted as DEV within the model statement, cause components that share the same model to vary independently of one another. Lot tolerances, denoted by LOT in the model statement, cause components that share a common model to vary together.

- If you want to apply only a DEV tolerance to a usual R, L or C device, you can edit the TOLERANCE attribute on the part.

- To add both LOT and DEV tolerance to R, L and C devices, you must use the breakout symbols Rbreak, Lbreak and Cbreak in place of the usual R, L and C parts.

2.1   Simulation Settings and Simulation Results

The simulation profile could be a DC Sweep analysis as follows:
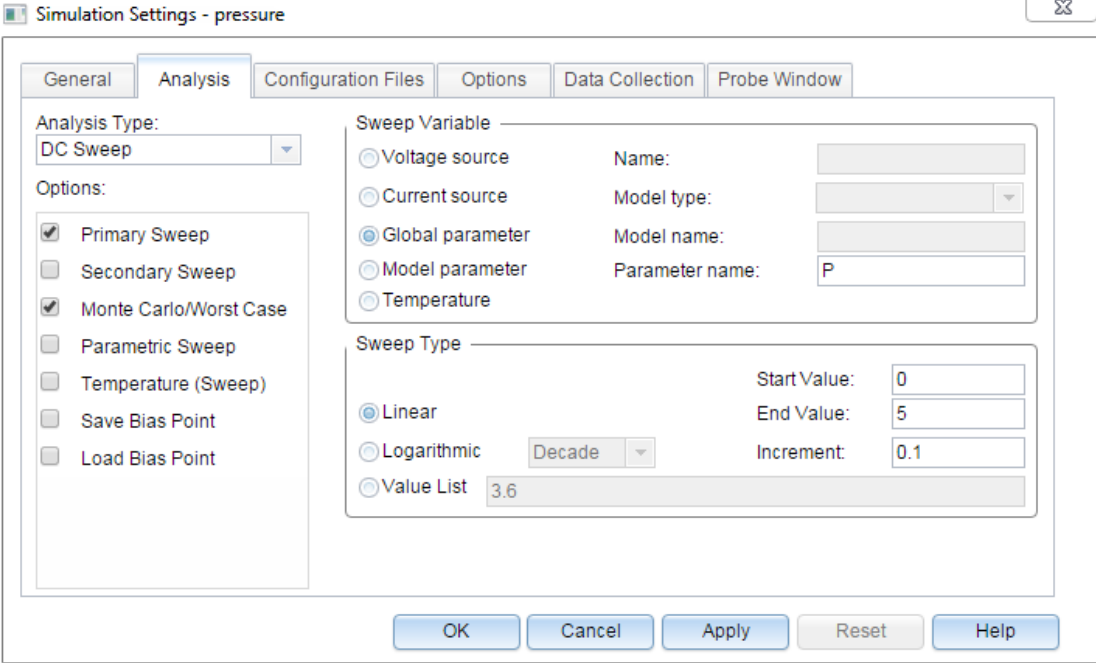
In Monte Carlo/Worst Case option, set I(Meter) as the Output variable, use Gaussian as distribution. If you left Random number seed blank, the default value 17533 will be used for the random number generator.

Note

- It is possible to set different distribution for different components but this needs to be done at model level. The distributions defined in the .model statement override the setting in the simulation setting options for MC analysis.

  For example:
  ```
  .model Rmc1 RES R=1 DEV/UNIFORM=2% LOT/UNIFORM=8%
  .model Rmc2 RES R=1 DEV/GAUSSIAN=2% LOT/GAUSSIAN=8%
  ```

- If you click **MC Load/Save,** the **Load/Save Monte Carlo Parameter File** dialog box comes up. In the text box that is enabled, specify the name and the location of the file in which the parameter data is to be saved. After running the Monte Carlo analysis, the model parameter values used for various simulation runs will be stored in the .mcp file. Of course, you can reuse these parameter values by using the **Load parameter values from file** option.
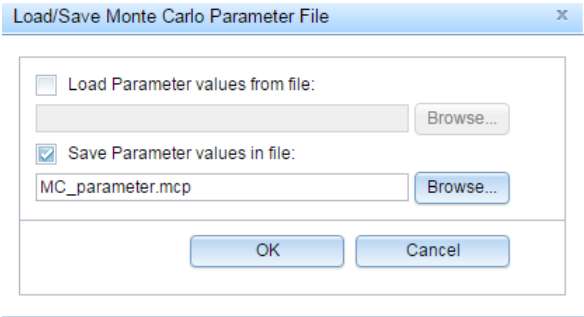
Click **More Settings** to choose a collating function e.g. YMAX. With the collating function you can define on which criteria the results shall be sorted in the output File You can check the **List model parameter values in the output file for each run** if you like.
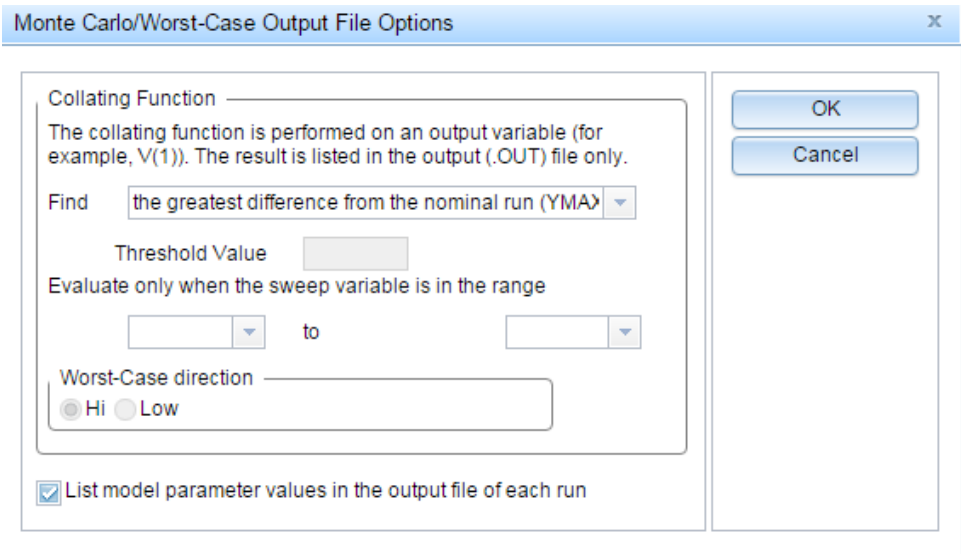
After the simulation the curves of I(Meter) could be as follows:
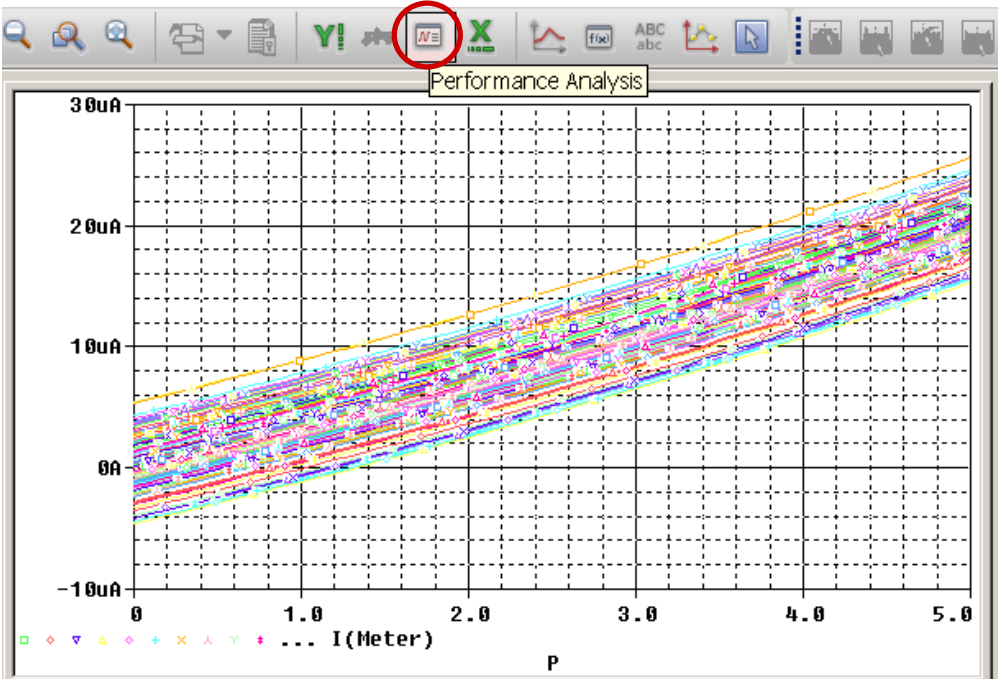
Note

With many runs (100 runs in our case), the waveform display can look more like a band than a set of individual waveforms. This can be useful for seeing the typical spread for a particular output variable. As the number of runs increases, the spread more closely approximates the actual worst-case limits for the circuit.

Click the **Performance Analysis** button, add trace Max(I(Meter)) to display histograms.

n samples	= 100	10th %ile	= 1.71752e-005
n divisions	= 10	median	= 2.00674e-005
mean	= 2.01278e-005	90th %ile	= 2.35522e-005
sigma	= 2.38141e-006	maximum	= 2.56332e-005
minimum	= 1.53505e-005	3*sigma	= 7.14423e-006
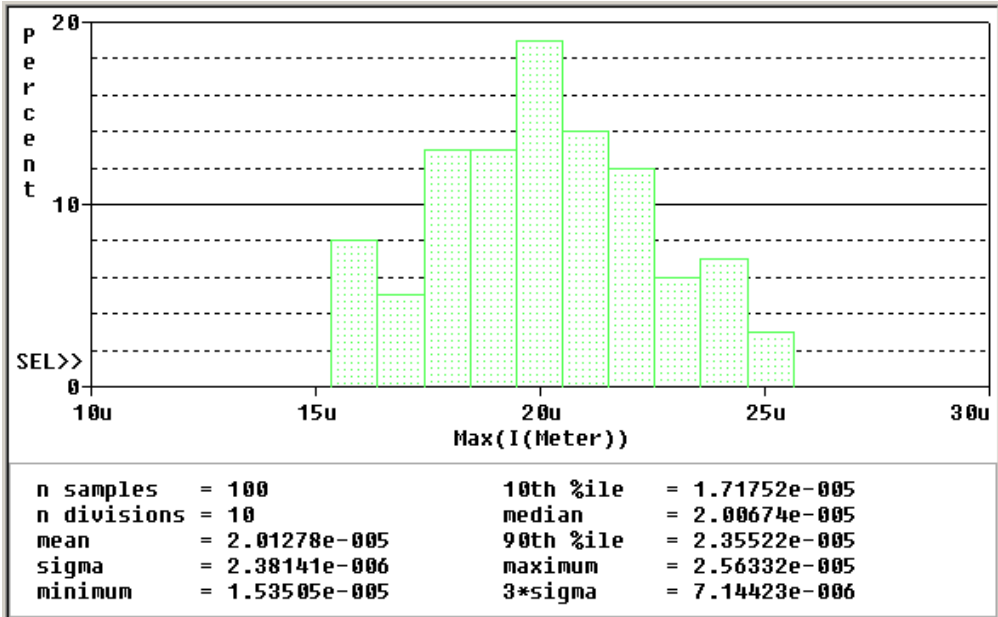
A MONTE CARLO SUMMARY is reported according to the output variable in simulation set up and the used collating function. The deviations are printed in order of size along with their run number.

The Quantity of the bars in the bar chart (histogram) can be set under **Tools > Options > Number of Histogram Divisions.**

```
****       SORTED DEVIATIONS OF I(V_Meter)  TEMPERATURE =   27.000 DEG C

                     MONTE CARLO SUMMARY

***************************************************************************


Mean Deviation =    28.5690E-09
Sigma          =     2.4445E-06

  RUN                      MAX DEVIATION FROM NOMINAL

Pass    28                 5.4934E-06  (2.25 sigma)  higher  at P =    3.6
                          ( 139.92% of Nominal)

Pass    91                 5.4736E-06  (2.24 sigma)  higher  at P =    4.8
                          ( 128.52% of Nominal)

Pass    64                 4.7988E-06  (1.96 sigma)  lower   at P =    5
                          (  76.184% of Nominal)

Pass    42                 4.5950E-06  (1.88 sigma)  lower   at P =    0
                          (  -8.4763E+15% of Nominal)
```

Fig. 3: MONTE CARLO SUMMARY with original Simulation Settings

Note

When you use the YMAX collating function, the output file also lists mean deviation and sigma values. These are based on the changes in the output variable from nominal at every sweep point in every Monte Carlo run.

2.2 MONTE CARLO SUMMARY

In the above diagram, the first **RUN** is **Pass 28,** because the deviation from nominal is the biggest in all of the Monte Carlo runs.

The maximal deviation is **5.4934E-06 (2.25 sigma),** because of 2.4445E-06*2.25≈5.500E-06.

Higher at P=3.6 means I(Meter) at the 28th run is bigger than at the nominal run while the sweep parameter P=3.6.

Note

There could be some rounding errors while calculating. The exact value is not very important here, the point is to understand the methodology within Monte Carlo analysis.

If you are interested which parameter values are used at pass 28, you can get the information in PSpice output file (the parameter values are also saved in MC_parameter.mcp).

```
                    MONTE CARLO PASS 28

************************************************************************


        **** CURRENT MODEL PARAMETERS FOR DEVICES REFERENCING Rmc
                         R_R1          R_R2       R_R3
           R          9.6564E-01    9.8941E-01   1.0152E+00


                      R_R_Pressure
           R          9.4454E-01
```
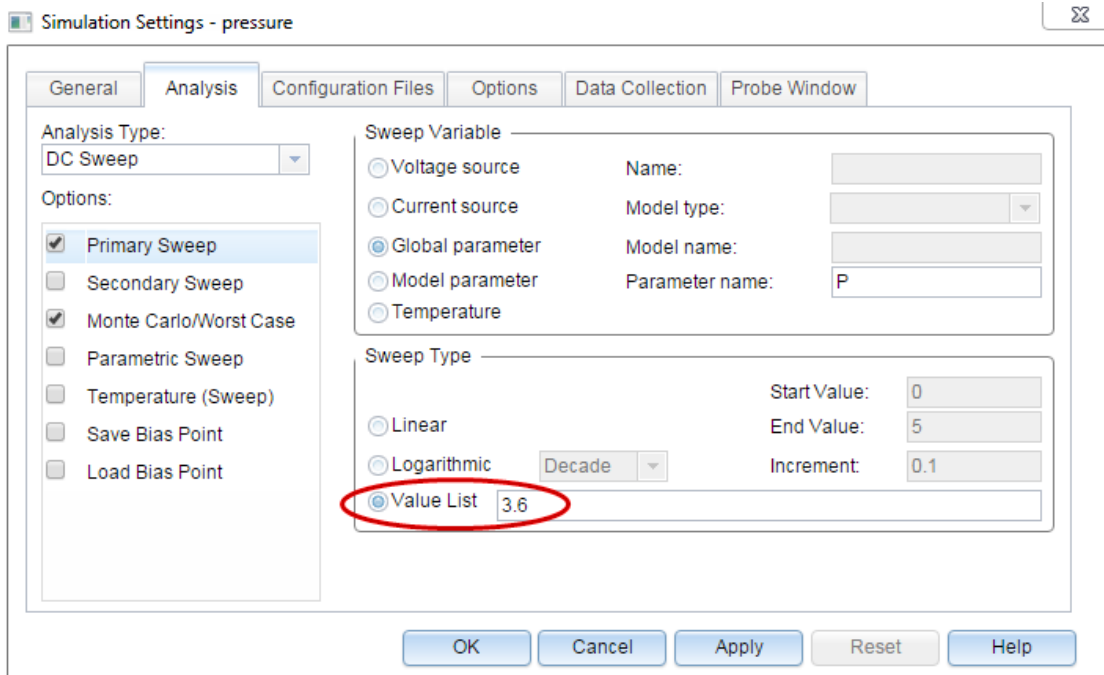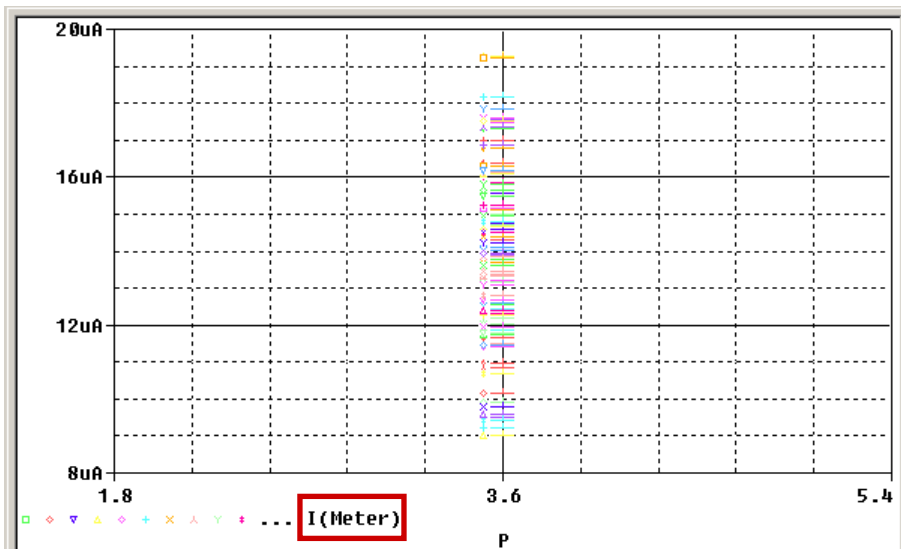
Now go back to **Simulation Settings** window and set the sweep variable P=3.6 and run the simulation again.

The new simulation results could be as follows:

Just marking **I(Meter) > Edit > Copy,** then open the Texteditor and paste the data, save the file as e.g. my_data.txt.

In this file you will get I(Meter)

- at the nominal run is 1.3759909052169e-005
- at the 28th run is 1.92532643268351e-005

Note

In MC analysis, the first run is always the nominal run.

Therefore, the difference is:
1.92532643268351e-005 - 1.3759909052169e-005 ≈ 5.4934e-006

Please look at the MAX DEVIATION FROM NOMINAL at Pass 28 in MONTE CARLO SUMMARY section in PSpice output file (see Fig. 3), it is **5.4934E-06** and the same as our calculated value.

There is still **139.92% of Nominal** because
1.92532643268351e-005/1.3759909052169e-005 ≈ 139.92%.

2.3 Histogram and Statistical Data

Here we would like to reduce the number of runs to have less data for better calculation. But in fact, you may need much more runs to get more reasonable statistical data.

The sweep variable P remains with the value 3.6 and we just reduce the number of runs from 100 to 10.
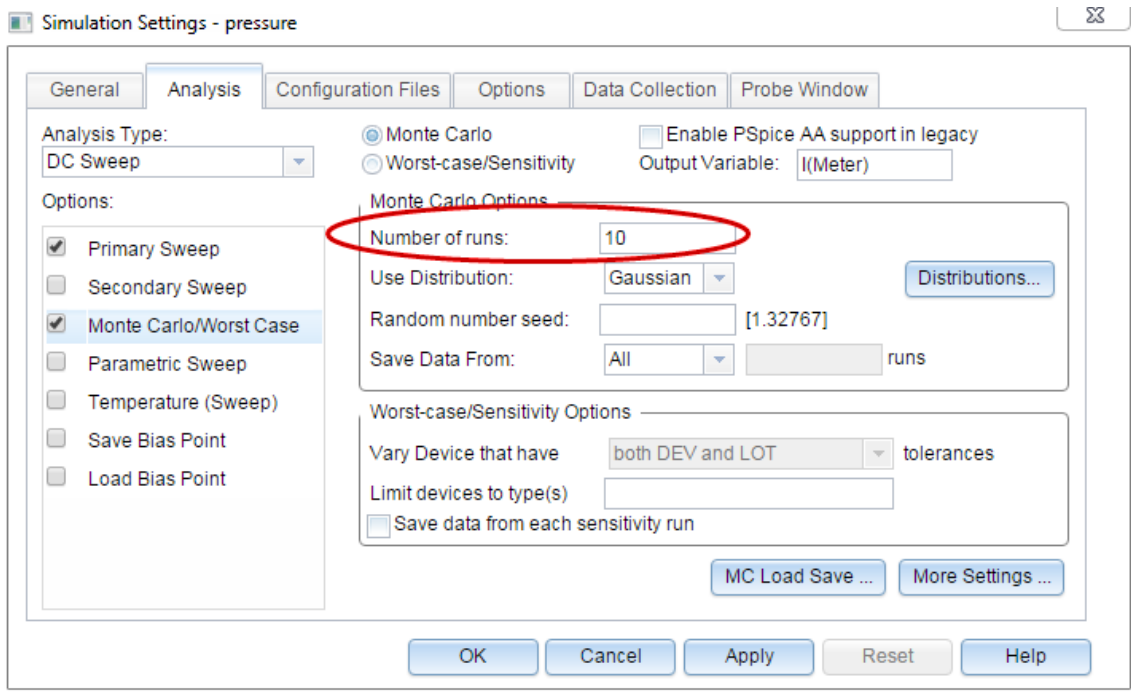
After the simulation we can display the histogram for I(Meter).
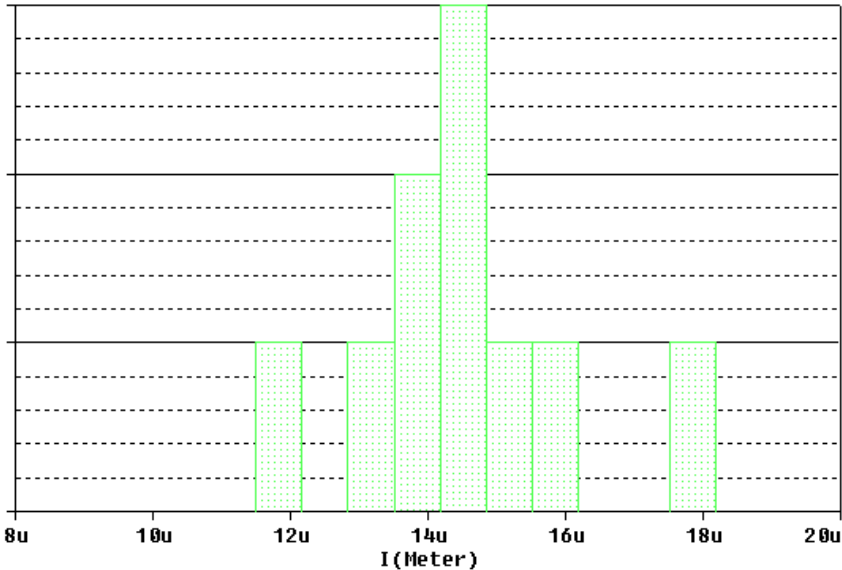
Fig. 4: Histogram of I(Meter)

The associated statistics are as follows:

```
n samples    = 10              10th %ile    = 1.23617e-005
n divisions  = 10              median       = 1.42536e-005
mean         = 1.44607e-005    90th %ile    = 1.71466e-005
sigma        = 1.7659e-006     maximum      = 1.81888e-005
minimum      = 1.15008e-005    3*sigma      = 5.29769e-006
```

n samples   = 10, because there are 10 runs
n divisions = 10, because the Number of Histogram Divisions is 10
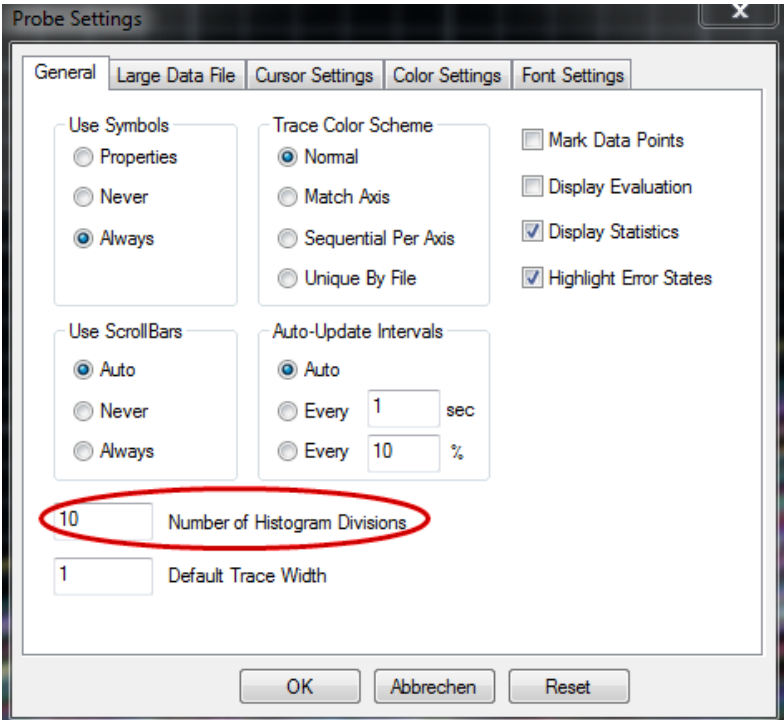
mean   = 1.44607e-005

The values of I(Meter) at simulation runs are listed in the table below (my_data_10runs.txt).

Section Number	I(Meter)
1	1.3759909052169e-005
2	1.43037614179775e-005
3	1.39189451147104e-005
4	1.61043353728019e-005
5	1.32225886773085e-005
6	1.81887880899012e-005
7	1.44032792377402e-005
8	1.1500794244057e-005
9	1.50016412590048e-005
10	1.42034468808561e-005

The mean μ is defined as

$$\mu = \frac{1}{n} \bullet \sum_{i=1}^{n} x_i$$

In this case
mean = (I(Meter)@1+I(Meter)@2+…+I(Meter)@10)/10
        = (1.3759909052169e-005+…+1.42034468808561e-005)/10 ≈ 1.44607e-005

sigma = 1.7659e-006

The standard deviation σ is defined as

$$\sigma = \sqrt{E[(X-\mu)^2]}$$

Here

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$= \sqrt{\frac{1}{10-1}[(I(Meter)@1-\mu)^2+...+(I(Meter)@10-\mu)2]}$$

    ≈ 1.7659e-006

minimum = 1.15008e-005 at 8th run

maximum = 1.81888e-005 at 6th run

10th %ile = 1.23617e-005

10% goal function values (I(Meter)) are less than or equal 1.23617e-005, and 90% goal function values are greater than or equal 1.23617e-005.

$$\tilde{x}_p = \frac{1}{2}\left(x_{n\bullet p} + x_{n\bullet p+1}\right),$$ if n●p integral,

$$\tilde{x}_p = x\lceil n \bullet p \rceil,$$ if n●p not integral

In our case n=10, p=10% → n●p=1, integral

$$\tilde{x}_p = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(I(Meter)@8 + I(Meter)@5\right)$$

$\approx 1.23617e{-}005$

Please see the file quantile.txt for the modified order of samples.

```
quantile.txt - Editor                                  _ □ ×
Datei  Bearbeiten  Format  Ansicht  ?

Section Number   I(Meter)                xi
8                1.1500794244057e-005    x1
5                1.32225886773085e-005   x2
1                1.37599909052169e-005   x3
3                1.39189451147104e-005   x4
10               1.42034468808561e-005   x5
2                1.43037614179775e-005   x6
7                1.44032792377402e-005   x7
5                1.50016412590048e-005   x8
4                1.61043353728019e-005   x9
6                1.81887880899012e-005   x10
```

90th %ile = 1.71466e-005

90% goal function values (I(Meter)) are less than or equal 1.71466e-005, and 10% goal function values are greater than or equal 1.71466e-005.

median   = 1.42536e-005

$$\tilde{x} = x_{\frac{n+1}{2}},$$ if n odd

$$\tilde{x} = \frac{1}{2}\left(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}\right),$$ if n even

In this case, n = 10 and even

$$\tilde{x} = \frac{1}{2}(x_5 + x_6) = \frac{1}{2}\left(I(Meter)@10 + I(Meter)@2\right)$$

$= 1.42536e{-}005$

3*sigma   = 5.29769e-006 simply because sigma = 1.7659e-006

2.4 Difference of the Statistical Data Reported in Histogram and in Output File

The statistical data in histogram are for the samples measured using a function, e.g. Max(I(Meter)) in Fig. 1 or using a simulation output variable in **Add Traces** window, e.g. I(Meter) in Fig. 4. A histogram has nothing to do with the collating function.

The statistical data in output file are based on the output variable defined in Simulation Settings and the collating function. For example, the MONTE CARLO SUMMARY looks as follows:

```
                        MONTE CARLO SUMMARY

*****************************************************************************

Mean Deviation =   778.7100E-09
Sigma          =     1.7486E-06

  RUN                    MAX DEVIATION FROM NOMINAL

Pass      6               4.4289E-06  (2.53 sigma)  higher  at P =    3.6
                          ( 132.19% of Nominal)

Pass      4               2.3444E-06  (1.34 sigma)  higher  at P =    3.6
                          ( 117.04% of Nominal)

Pass      8               2.2591E-06  (1.29 sigma)  lower   at P =    3.6
                          (  83.582% of Nominal)

Pass      9               1.2417E-06  ( .71 sigma)  higher  at P =    3.6
                          ( 109.02% of Nominal)

Pass      7             643.3700E-09  ( .37 sigma)  higher  at P =    3.6
                          ( 104.68% of Nominal)

Pass      2             543.8500E-09  ( .31 sigma)  higher  at P =    3.6
                          ( 103.95% of Nominal)

Pass      5             537.3200E-09  ( .31 sigma)  lower   at P =    3.6
                          (  96.095% of Nominal)

Pass     10             443.5400E-09  ( .25 sigma)  higher  at P =    3.6
                          ( 103.22% of Nominal)

Pass      3             159.0400E-09  ( .09 sigma)  higher  at P =    3.6
                          ( 101.16% of Nominal)
```
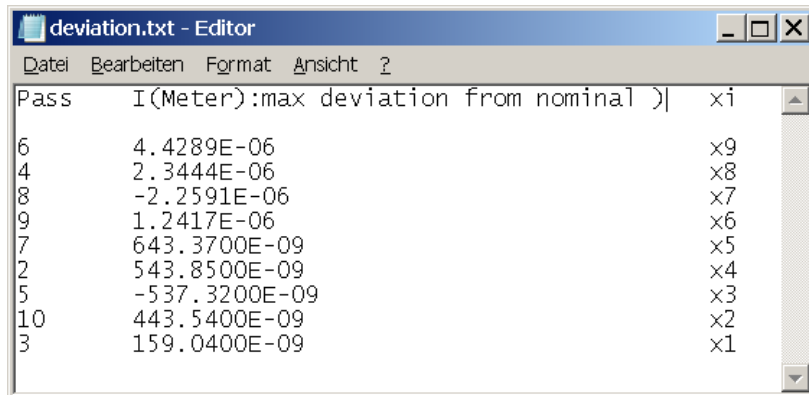
In this case there are 9 samples, not 10 samples. The sample value is the maximal deviation from nominal, not the value of a measurement or an available output variable.

Mean Deviation = 778.7100E-09

Here:

$$mean\ deviation = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\approx 778.71E\text{-}09$$

Please refer deviation.txt file which based on the data of MONTE CARLO SUMMARY. The values (x7 and x3) are negative, because they are lower than the nominal.

```
deviation.txt - Editor
Datei  Bearbeiten  Format  Ansicht  ?
Pass      I(Meter):max deviation from nominal )|    xi
6         4.4289E-06                                 x9
4         2.3444E-06                                 x8
8         -2.2591E-06                                x7
9         1.2417E-06                                 x6
7         643.3700E-09                               x5
2         543.8500E-09                               x4
5         -537.3200E-09                              x3
10        443.5400E-09                               x2
3         159.0400E-09                               x1
```

Sigma   = 1.7486E-06

In our case

$$Sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - Mean\ Deviation\right)^2}$$

$\approx 1.7486E-06$

3   Bibliography

[1]   PSpice User Guide, Cadence

[2]   PSpice Reference Guide, Cadence

[3]   OrCAD Capture User Guide, Cadence

[4]   http://de.wikipedia.org/wiki/Quantil

[5]   http://de.wikipedia.org/wiki/Standardabweichung